# A Fuzzy Set Semantics for Qualitative Fluents in the Situation Calculus

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**Abstract.** Specifying the behavior of an intelligent autonomous robot or agent is a non-trivial task. The question is: how can the knowledge of the domain expert be encoded in the agent program? Qualitative representations in general facilitate to express the knowledge of a domain expert. In this paper, we propose a semantics for qualitative fluents in the situation calculus. Our semantics is based on fuzzy sets. Membership functions define to which degree a qualitative fluent belongs to a particular category. Especially intriguing about a fuzzy set semantics for qualitative fluents is that the qualitative ranges may overlap, and a value can, at the same time, fall into several categories.

## 1 Introduction

Specifying the behavior of an intelligent autonomous robot or agent is a non-trivial task. Based on the available sensor values, usually a world model is constructed. With world model variables often a rule base is established which encodes the behavior of the agent. The question is: how can the knowledge of the domain expert be encoded in the agent program? One way to ease the specification problem for the domain axiomatizer is to make use of qualitative descriptions of the world. Humans are, in general, good in describing behaviors in qualitative terms. To this end, Zadeh for example introduced so-called linguistic variables in his pioneering work [1]. These variables allow an expert to state certain rules about an application domain in a demotic way, without having to care for tricky mathematical details and complex mathematical formalizations. Another facilitation is to use a behavior representation language which allows for flexible and expressive behavior specifications. Among the many different action formalisms, the situation calculus [2] is a popular one; it is a powerful calculus for reasoning about actions and change, but it is also popular as the successful robot programming and planning language Golog [3] is based on it.

In this paper we aim at combining both, an expressive representation language and the possibility to use a qualitative world model for facilitating the behavior specification. In our previous work [4], we proposed a qualitative world model for the robotic soccer application. However, the integration of the qualitative world model into our action formalism was ad hoc. In this paper, we put the original

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idea to a formal basis and propose a fuzzy set semantics for qualitative fluents. Our work is inspired by Dubois and Prade [5] who argue that "the main application [...] [of fuzzy logic was] fuzzy clustering and classification, [and] a smooth interface between numerical and symbolic knowledge [...]". We introduce a new type of qualitative fluents which have fuzzy membership functions associated. These functions describe to what extend a fluent value belongs to a particular qualitative category. Thus, we are able to pose qualitative queries of the form: *is obstacle close?* Especially intriguing about a fuzzy set semantics for qualitative fluents is that the ranges of the qualitative predicates may overlap, and a value can, at the same time, fall into several categories.

This paper is organized as follows. In Sect. 2 we discuss some related work reviewing the connection between qualitative representations and fuzzy logic. Sect. 3 gives the mathematical background: Sect. 3.1 briefly goes over the situation calculus, while we sketch the fundamental ideas of fuzzy membership functions in Sect. 3.2. In Sect. 4 we introduce fuzzy fluents, define fuzzy sets in the situation calculus as well as a membership relation for fuzzy fluents. In an extended example we show how fuzzy fluents can be applied for describing the world in a qualitative fashion. We conclude with Sect. 5.

## 2 Related Work

According to [5], "fuzzy sets and possibility theory offer a unified framework for taking into account the gradual or flexible nature of many predicates, requirements, and the representation of incomplete information". It is suited to represent human-like taxonomies in pattern classification, or lexical imprecision of natural language. The fuzzy membership function can be seen as a degree of similarity, degree of preference, or degree of plausibility. From the first interpretation the rough set theory evolved, while most engineering applications make use of fuzzy rules of the form $\mathit{if}\ X_1\ \mathit{is}\ A_1\ \mathit{and} \ldots X_n\ \mathit{is}\ A_n\ \mathit{then}\ Y\ \mathit{is}\ B$.

In [6], Bolloju follows similar ideas to use qualitative variables based on fuzzy sets for facilitating decision making. The knowledge used for problem-solving is structured hierarchically with so-called decision models at top and decision rules at the bottom layer. Decisions at the top layer are made by sequential, conditional, or parallel composition of decision rules of a lower layer. While the approach of Bolloju is similar to ours, his approach is very restricted in the way decisions can be made. Similar lines as to use fuzzy qualitative variables are followed in [7,8]. In our approach however, we can make use of the full expressiveness of the situation calculus for taking decisions. A very related approach is [9]. The authors propose a fuzzy test-score semantics for soft constraints. They propose a function *fholds* which evaluates the result of combinations of soft constraints in allusion to the *holds* predicate known from action logics. Although they state using the situation calculus, besides the function *fholds* they do not give a full axiomatization.

In an earlier work, Sugeno and Yasukawa [10] discussed the adequacy of fuzzy logic-based approaches for qualitative modeling. In their work they describe the process of generating a qualitative model based on fuzzy representations from a set of sample input-output data describing the system behavior. Based on heuristics they identify the input data which influence the output. Then, they approximate the number of fuzzy rules needed to describe the output data. The result is a fuzzy controller, i.e. a set of fuzzy rules, which describes the mapping from the given input values to the output values. Later, Tikk et al. [11] improved the original idea and proposed several algorithms for building trapezoid approximations of membership functions and for rule base reduction. Here, fuzzy logics is used for approximating a given system in a qualitative way.

As an example for the large body of work dealing with fuzzy control and robots, we want to mention the work of [12,13]. Soffiotti [12] shows how fuzzy controllers can be used to design robust behavior-producing modules, and even how high-level reasoning and low-level execution can be integrated on a mobile robot. He attributes the success of fuzzy logic in control to "its ability to represent both the symbolical and the numerical aspects of reasoning. Fuzzy logic can be embedded in a full logical formalism, endowed with a symbolic reasoning mechanism; but it is also capable of representing and processing numerical data." Liu et al. [13] describe a robot kinematics in a qualitative fashion making use of fuzzy descriptions. They use fuzzy qualitative trigonometric functions to describe the movements of a PUMA robot manipulator. This fuzzy qualitative description is according to the authors very helpful for calibration procedures in terms of measuring accuracy or repeatability. They further stress that the fuzzy qualitative predicates provide the connection between the numerical data and interval symbols, which are then used for building up the behavior vocabulary from which in turn the motion control of the robot can be derived. Many other successful examples for fuzzy control applications are given in [14]. Good overviews of the fields are also given in [5,15,16,17].

## 3 The Situation Calculus and Fuzzy Sets

### 3.1 The Situation Calculus

The situation calculus is a second order language with equality which allows for reasoning about actions and their effects. The world evolves from an initial situation due to primitive actions. Possible world histories are represented by sequences of actions. The situation calculus distinguishes three different sorts: *actions*, *situations*, and domain dependent *objects*. A special binary function symbol $do : action \times situation \rightarrow situation$ exists, with $do(a, s)$ denoting the situation which arises after performing action $a$ in the situation $s$. The constant $S_0$ denotes the initial situation, i.e. the situation where no actions have occurred yet. The state the world is in is characterized by functions and relations with a situation as their last argument. They are called *functional* and *relational fluents*, resp. As an example consider the position of a robot navigating in an office environment. One aspect of the world state is the robot's location $robotLoc(s)$. Suppose the robot is in an office with room number 6214 in the initial situation $S_0$. The robot now travels to office 6215. The position of the robot then changes to $robotLoc(do(goto(6215), S_0)) = 6215$. $goto(6215)$ denotes the robot's action of traveling from office 6214 to 6215. The situation the world is in is described by $s_1 = do(goto(6215), S_0)$. The value of the functional fluent $robotLoc(s_1)$ equals 6215. The third sort of the situation calculus is the sort *action*. Actions are characterized by unique names. For each action one has to specify a *precondition axiom* stating under which conditions it is possible to perform the respective action and an *effect axiom* formulating how the action changes the world in terms of the specified fluents.

An action precondition axiom has the form $Poss(a(\boldsymbol{x}), s) \equiv \Psi(\boldsymbol{x}, s)$ where the binary predicate $Poss \subseteq action \times situation$ denotes when an action can be executed, and $\boldsymbol{x}$ stands for the arguments of action $a$. For our travel action the precondition axiom may be $Poss(goto(room), s) \equiv robotLoc(s) \neq room$. After having specified when it is physically possible to perform an action it remains to be detailed how the respective action changes the world. In the situation calculus the effects of actions are formalized by so-called successor state axioms of the form $F(\boldsymbol{x}, do(a, s)) \equiv \varphi_F^+(\boldsymbol{x}, a, s) \vee F(\boldsymbol{x}, s) \wedge \neg\varphi_F^-(\boldsymbol{x}, a, s)$, where $F$ denotes a fluent, $\varphi_F^+$ and $\varphi_F^-$ are formulas describing under which conditions $F$ is true, or false resp. This axiom simply states that $F$ is true after performing action $a$ if $\varphi_F^+$ holds, or the fluent keeps its former value if it was not made false. Successor state axioms describe Reiter's solution to the frame problem [18], the problem that all the non-effects of an action have to be formalized as well. The background theory is a set of sentences $\mathcal{D}$ consisting of $\mathcal{D} = \Sigma \cup \mathcal{D}_{ssa} \cup \mathcal{D}_{ap} \cup \mathcal{D}_{una} \cup \mathcal{D}_{S_0}$, where $\mathcal{D}_{ssa}$ contains sentences about the successor state axioms, $\mathcal{D}_{ap}$ contains the action precondition axioms, $\mathcal{D}_{una}$ states sentences about unique names for actions, and $\mathcal{D}_{S_0}$ consists of axioms what holds in the initial situation. Additionally, $\Sigma$ contains a number of foundational axioms defining situations. For details we refer to [19,18].

### 3.2 Membership Functions and Fuzzy Sets

A *crisp set* $A$ over a universe of discourse $U$ can be defined by $A = \{x | x$ meets some condition$\}$ and be stated with a subset relation if any $x \in U \subset A$; or alternatively define a characteristic function $\mu_A$ as $\mu_A = 1$, if $x \in A$, and $\mu_A = 0$, otherwise. For two sets $A, B \subset U$, the union $A \cup B$ is defined as $\mu_{A\cup B}(x) = 1$ if $x \in A$ or $x \in B$ and $\mu_{A\cup B}(x) = 0$ if $x \notin A$ and $x \notin B$. For the intersection $A \cap B$ it holds: $\mu_{A\cap B} = 1$ if $x \in A$ and $x \in B$, and $\mu_{A\cap B} = 0$ if $x \notin A$ or $x \notin B$. The complement $\bar{A}$ is defined such that $\mu_{\bar{A}}(x) = 1$ if $x \notin A$; $\mu_{\bar{A}}(x) = 0$ if $x \in A$. Following [16], it can be shown that

$$A \cup B \supset \mu_{A\cup B}(x) = \max[\mu_A(x), \mu_B(x)] \quad (1)$$

$$A \cap B \supset \mu_{A\cap B}(x) = \min[\mu_A(x), \mu_B(x)] \quad (2)$$

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \quad (3)$$

Crisp set operations enjoy the property of being commutative, associative, and distributive. Further, De Morgan's laws as well as the Law of Contradiction and Excluded Middle hold.

A *fuzzy set* $F$ with a universe of discourse $U$, on the other hand, can only be characterized with help of a membership function $\mu_F : U \to [0,1]$. The membership function provides a measure of the degree of similarity of an element in $U$ to the fuzzy set. A fuzzy set $F$ in $U$ can be represented as a set of ordered pairs of a generic element $x$ and its grade of membership: $F = \{(x, \mu_F(x)) | x \in U\}$. Union, intersection, and complement can be defined the same way as for crisp sets (Eq. 1–3). Note that unlike for crisp sets, the Law of Contradiction and the Excluded Middle do *not* hold, i.e. $A \cup \bar{A} \neq U$ and $A \cap \bar{A} \neq \emptyset$. In the definitions of intersection and union we used the "max" and "min" operators. But these are not the only possibilities to define fuzzy union and fuzzy intersection. In general, for set intersection several different so-called t-norms have been proposed, usually written as $\mu_{R\cap S}(x) = \mu_R(x) \star \mu_S(x)$, for set union t-conorms or s-norms, written as $\mu_{R\cup S}(x) = \mu_R(x) \oplus \mu_S(x)$ were formulated (see e.g. [5] for examples). Here, we rely on the min t-norm (Eq. 1), the max t-conorm (Eq. 2) and Eq. 3 for the complement.

To be able to conclude something useful with fuzzy variables, we need inference rules which define how to reason with variables of this kind. In the following, we give several examples of the type of inference possible by stating examples from [15]: (1) *Categorical rules* like *X is small*; (2) *Entailment rules* like *Mary is very young* and *very young implies young* implies *Mary is young*; (3) *Conjunction/Disjunction rules* like *pressure is not very high* and/or *pressure is not very low* implies *pressure is not very high and/or not very low*; (4) *Compositional rules* like *X is much larger than Y* and *Y is large* implies *X is much larger* $\circ$ *large*; (5) *Negation rules* like *not(Mary is young)* implies *Mary is not young*; (6) *Extension principle* like *X is small* implies $X^2$ *is* $^2$*small* with $^2$small meaning *very small*. Another way to see these rules is by interpreting $X$ and $Y$ as decision variables and $A$ as a soft constraint, allowing for a degree of membership. If there are only two membership values (true or false), then these constraints can be regarded as hard constraints (see e.g. [5]).

## 4 Membership Values for Qualitative Fluents

In this section, we introduce our membership semantics to situation calculus fluents. We start with restating our example from [4], now with the new membership semantics as a motivating example. Then, we formally introduce fuzzy sets, fuzzy fluents, and the membership relation. Finally, we give an extended example of the new semantics of qualitative fluents.

### 4.1 A Motivating Example

Consider the following soccer example. Our qualitative world model consists of different predicates describing the position on the field as well as the orientation. The position is defined by several equivalence classes for $(x, y)$ positions. The classes we defined here are *zoneFarBack*, *zoneBack*, *zoneMiddle*, *zoneFront*, and *zoneFarFront* for the $x$ coordinate, and *sideLeft*, *sideMiddle*,

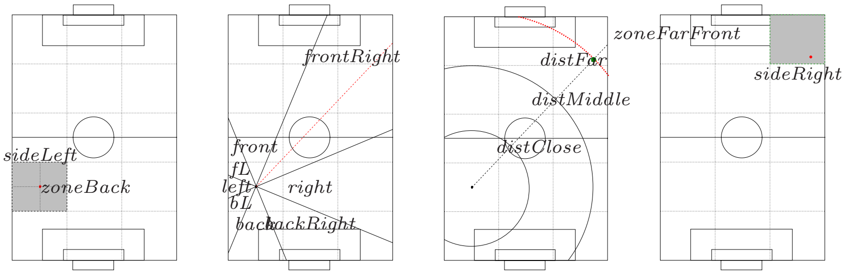



**Fig. 1.** A Soccer Example

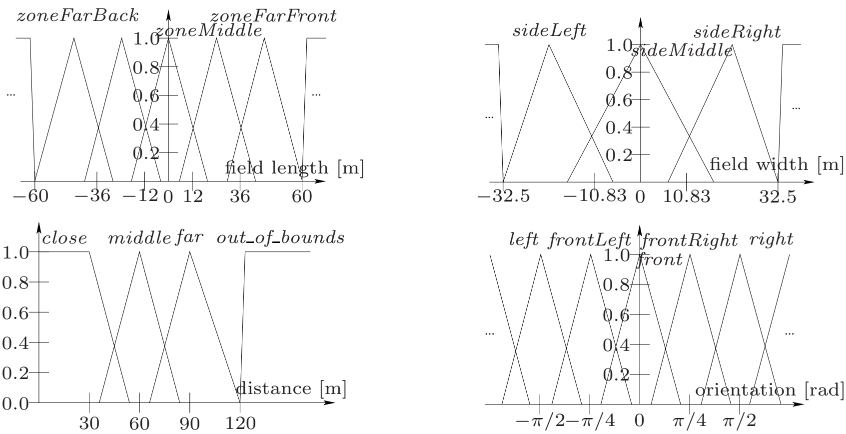



**Fig. 2.** Distance and Orientation

and *sideRight* for the $y$ coordinate. The origin is in the center of the field with the positive $x$ axis pointing northwards, and the positive $y$ axis pointing to the west. The orientation of the robot is given by the qualitative categories $\ldots, \mathit{left}, \mathit{frontLeft}, \mathit{front}, \mathit{frontRight}, \mathit{right}, \ldots$. Suppose that, in situation $s_n$ the robot is located at the qualitative pose $\mathit{pose}(s_n) = (\mathit{zoneBack}, \mathit{sideLeft}, \mathit{front})$. Suppose further, that in situation $s_n$ the high-level control program of the agent contains the action goto_relative($\mathit{far}, \mathit{frontRight}$), mentioning the qualitative categories *far* and *frontRight*. The question is, at which position will the robot end up in the situation $s_{n+1} = \mathit{do}(\text{goto\_relative}(\mathit{far}, \mathit{frontRight}), s_n)$?

As one can see in Fig. 1, the result should be that the agent ends up at pose $\mathit{pose}(s_{n+1}) = (\mathit{zoneFront}, \mathit{sideRight}, \mathit{frontRight})$. Fig. 2 shows possible membership functions suitable for the soccer domain. Assume the following successor state axiom for the pose fluent:

$$
\begin{aligned}
&\mathit{pose}(\mathit{do}(a, s)) = (x, y, \theta) \equiv \\
&\quad \ldots a = \text{goto\_relative}(d, \psi) \wedge \mathit{pose}(s) = (x', y', \theta') \wedge \\
&\qquad \theta = \theta' + \psi \wedge x = x' + d \cdot \sin\theta \wedge y = y' + \cos\theta \ldots
\end{aligned}
$$

In order to calculate the new pose of the robot after performing the action, we have to defuzzify the qualitative predicate values according to the membership of the qualitative category mentioned. The literature on fuzzy logic mentions several defuzzifier. Here, we choose the center of gravity, which we give in Definition 5. As for now, it is sufficient to know that it returns the abscissa of the centroid of the respective category. In the case of the triangular-shaped membership functions we used here, this is simply the apex of the membership function. For our example, this means that the pose in situation $s_n$ is mapped to the numerical values $(x, y, \theta)^T = (-24, -21.6, 0)^T$. Now, for applying the action goto_relative(*far*, *frontRight*) we need to defuzzify the qualitative distance and orientation *far* and *frontRight* to 90 and $\pi/4$. Applying the successor state axiom yields the new quantitative pose $pose(s_{n+1}) = (52.58, 25.67, \pi/4)$. Applying the membership function for the qualitative position and orientation, yields a new qualitative pose $pose(s_{n+1}) = (zoneFarFront, sideRight, frontRight)$.

### 4.2 Fuzzy Sets in the Situation Calculus

As we have seen in the previous section, using fuzzy membership functions facilitates the representation of qualitative world model predicates. The reason is that with fuzzy representations using membership functions, one has a means to access and alter the abstraction from quantitative to qualitative predicates, and using defuzzifiers, one can easily associate a qualitative category with a single distinct quantitative value.

In the following, we elaborate on this idea and extend the situation calculus in such a way that qualitative world model predicates can be described in terms of membership values. To ease the presentation, we assume a discrete representation of our membership functions.

**Definition 1 (Reals and Linguistic Terms).** *We introduce two new sorts to the situation calculus: real and linguistic. We do not axiomatize reals here, and assume their standard interpretation together with the usual operations and ordering relations. Linguistic terms are a finite set of constant symbols* $c_1, \ldots, c_k$ *in the language. They refer to qualitative classes; examples are* close *or* far*. We further require a unique names assumption for these linguistic categories, i.e.* $\forall i, j, c_i, c_j . i \neq j \supset c_i \neq c_j$.

Now, having introduced reals and linguistic terms into the language of the situation calculus, we can define the degree of membership of a particular value to a given category. For ease of notation we assume that the domain of a particular category is from the domain of real numbers. In general, the domain can be defined arbitrarily.

**Definition 2 (Fuzzy Sets).** *Let* $c_1, \ldots, c_k$ *be categories of sort linguistic. We introduce a relation* $\mathfrak{F} \subseteq linguistic \times real \times [0, 1]$ *relating each linguistic term* $c$ *of the domain, a real number, and a degree of membership in the category* $c$ *as*

$$\begin{aligned}&\forall c, u, \mu_u . \mathfrak{F}(c, u, \mu_u) \equiv \\ &\quad (c = c_1 \supset u = u_{c_1,0} \wedge \mu_u = \mu_{c_1,0} \vee \cdots \vee u = u_{c_1,m_1} \wedge \mu_u = \mu_{c_1,m_1}) \vee \ldots \vee \\ &\quad (c = c_k \supset u = u_{c_k,0} \wedge \mu_u = \mu_{c_k,0} \vee \cdots \vee u = u_{c_k,m_k} \wedge \mu_u = \mu_{c_k,m_k}),\end{aligned}$$

*where all* $u_{c_i,j}$ *and* $\mu_{c_i,j}$ *are constants of sort real and* $\mu_{c_i,j} \in [0,1]$ *respectively, i.e.* $\forall c, u, \mu_u.\mathfrak{F}(c,u,\mu_u) \supset 0 \le \mu_u \le 1$. *To ensure that, for each category, each pair* $(u, \mu_u)$ *is unique, we require* $\Sigma$ *to contain the constraint* $\forall c \exists u, \mu_u \forall \mu_{u'}.\mathfrak{F}(c,u,\mu_u) \land \mathfrak{F}(c,u,\mu_{u'}) \supset \mu_u = \mu_{u'}$. *We further require one of the* $u_{c_i,j}$ *to equal the center of gravity of the respective category, i.e.* $u_{c_i,j} = cog(c_i)$ *(cf. Eq. 4 in Def. 5).*

Note that the above definition yields a formalization of fuzzy sets $F = \{(x, \mu(x)) | x \in U\}$ as described in Sect. 3.2. A common notation in the fuzzy control literature is $F = \sum_U \mu_F(x)/x,\ x \in U$. Further note that variables occurring free in the logical sentences are implicitly universally quantified in the following definitions. As we restricted $\mathfrak{F}$ to assign membership values ranging from 0 to 1 to real numbers, we need to ensure that the fluent to which we apply qualitative categories, takes only real values. Therefore, we need to introduce fuzzy fluents as a specialization of functional fluents.

**Definition 3 (Fuzzy Fluent).** *A fuzzy fluent* $\mathfrak{f}$ *is a functional fluent restricted to take only values from sort real. We write* $\mathfrak{f}(\boldsymbol{x}, s)$ *to refer to a fuzzy fluent, and* $f(\boldsymbol{x}, s)$ *to refer to a non-fuzzy fluent.*

To query whether or not a fluent value belongs to a certain category, we introduce, similar to fuzzy control theory, predicates $is_{\ldots}$. These predicates are true if a fuzzy fluent value belongs to the category in question to a non-zero degree.

**Definition 4 (Membership).**

1. *To query if a fuzzy fluent belongs to a given category, we define the predicate* is $\subseteq$ *real* $\times$ *linguistic as*

$$\text{is}(\mathfrak{f}(\boldsymbol{t}, \theta), \gamma) \doteq \exists u, \mu_u.\mathfrak{f}(\boldsymbol{t}, \theta) = u \land \mathfrak{F}(\gamma, u, \mu_u) \land \mu_u > 0.$$

2. *Similarly, we define* $is_{\complement} \subseteq$ *real* $\times$ *linguistic, to know if a fuzzy fluent does* not *belong to a certain category*

$$\text{is}_{\complement}(\mathfrak{f}(\boldsymbol{t}, \theta), \gamma) \doteq \neg \exists u, \mu_u.\mathfrak{f}(\boldsymbol{t}, \theta) = u \land \mathfrak{F}(\gamma, u, \mu_u) \lor \exists u, \mu_u.\mathfrak{f}(\boldsymbol{t}, \theta) = u \land \mathfrak{F}(\gamma, u, \mu_u) \land \mu_u < 1.$$

   *It holds that a fluent value does not belong to a certain category, if either the value in question is not defined in terms of a fuzzy set, or the value exists and its degree of membership is less than 1 (cf. also Eq. 3 for the definition of complement).*

3. *For complex queries, for example if a fuzzy fluent value belongs to several overlapping categories at the same time, we define a predicate* $\text{is}_\star \subseteq$ *real* $\times$ $(linguistic)^n$ *with a finite* $n \in \mathbb{N}$ *as*

$$\text{is}_\star(\mathfrak{f}(\boldsymbol{t}, \theta), \gamma_0, \ldots, \gamma_n) \doteq \exists u, \mu_{u,0}, \ldots, \mu_{u,n}.\mathfrak{f}(\boldsymbol{t}, \theta) = u \land \mathfrak{F}(\gamma_0, u, \mu_{u,0}) \land \cdots \land \mathfrak{F}(\gamma_n, u, \mu_{u,n}) \land (\mu_{u,0} \star \cdots \star \mu_{u,n} > 0).$$

   *Here, the t-norm* $\star$ *refers to the min operation as given in Eq. 2.*

4. *Similarly, for asking whether or not a fuzzy fluent value belongs to one category or the other, we introduce the predicate* $is_\oplus : real \times (linguistic)^n$ *with a finite* $n \in \mathbb{N}$

$$\mathrm{is}_\oplus(\mathfrak{f}(\boldsymbol{t}, \theta), \gamma_0, \ldots, \gamma_n) \doteq \exists u, \mu_{u,0}, \ldots, \mu_{u,n}.\mathfrak{f}(\boldsymbol{t}, \theta) = u \wedge \mathfrak{F}(\gamma_0, u, \mu_{u,0}) \\ \wedge \cdots \wedge \mathfrak{F}(\gamma_n, u, \mu_{u,n}) \wedge (\mu_{u,0} \oplus \cdots \oplus \mu_{u,n} > 0).$$

*In our case, the s-norm* $\oplus$ *refers to the max-operator as given in Eq. 1.*

By now, we defined fuzzy fluents as a specialization of functional fluents operating on reals, introduced qualitative categories as constants of sort *linguistic*, and defined a fuzzy set in our domain axiomatization which allows for defining which values make up a qualitative category to which degree. We can further query, whether or not a fuzzy fluent belongs to a qualitative category. Moreover, we can ask, if a fuzzy fluent belongs to several categories at the same time, or if it belongs to the complementary category. What is still missing, though, is to assign a qualitative value to a fuzzy fluent.

**Definition 5 (Assignment of Fuzzy Category Values).** *For a fuzzy fluent* $\mathfrak{f}(\boldsymbol{x}, s)$ *we define a special assignment operator* $:=$ *which assigns the value of a category* $c$ *to the fluent* $\mathfrak{f}$ *in situation* $s$. *The intention is to assign the category's mean value* $\hat{u}$ *with*

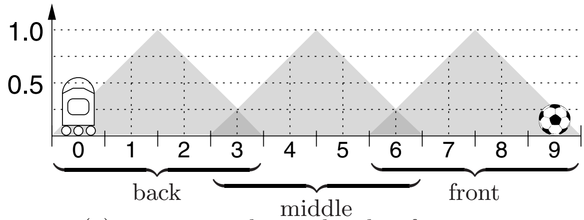
$$\begin{aligned} & cog(c) = \hat{u} \equiv \\ & \exists u_0, \ldots, u_k, \mu_{u_0}, \ldots, \mu_{u_k}.\mathfrak{F}(c, u_0, \mu_{u_0}) \wedge \cdots \wedge \mathfrak{F}(c, u_k, \mu_{u_k}) \wedge \\ & \quad u_0 \neq \cdots \neq u_k \wedge \forall u^*, \mu^*.(u^* \neq u_0 \wedge \cdots \wedge u^* \neq u_k \wedge \\ & \quad\quad \mu^* \neq \mu_{u_0} \wedge \cdots \wedge \mu^* \neq \mu_{u_k} \supset \neg\mathfrak{F}(c, u^*, \mu^*)) \wedge \\ & \quad \hat{u} = \sum_{i=1}^{k} u_i \cdot \mu_{u_i} \Big/ \sum_{i=1}^{k} \mu_{u_i} \end{aligned} \tag{4}$$

*denoting the center of gravity of all values defining the category* $c$. *The assignment action can be formalized by adding the following case to the successor state axiom of fluent* $\mathfrak{f}$: $\mathfrak{f}(\boldsymbol{x}, do(a, s)) = \hat{u} \equiv \ldots a = :=(c) \wedge cog(c) = \hat{u} \ldots.$

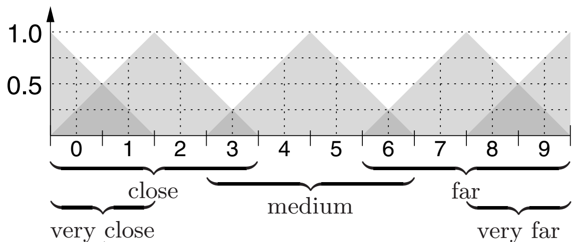
We have now defined everything we need to reason with qualitative predicates based on fuzzy membership functions. In the next section we illustrate it with an example.

### 4.3 Reasoning with Qualitative Categories

Consider the following simple example we use to illustrate qualitative reasoning with linguistic terms introduced above. A robot is situated in a one dimensional domain. The domain consists of 10 metric units which are partitioned in three categories, namely *back*, *middle*, and *front*. Distance is also partitioned in categories, namely *close*, *medium*, and *far*. We give the (fuzzy) definition of those categories below, where we use $(u_i, \mu_i)$ as an abbreviation for $u = u_i \wedge \mu = \mu_i$. To keep things simple, we restrict ourselves to integer values for positions. The




(a) position with membership functions




(b) distance with membership functions

**Fig. 3.** A simple one dimensional robot domain

robot can move forward in integer steps. Restricting to integers presupposes that we need to use an altered version $cog'(c)$ of the center of gravity formula: $cog'(c) \doteq \lfloor cog(c) \rfloor$. The fuzzy categories for the position of the robot in the world can be defined as

$$\begin{aligned}&\mathfrak{F}(position, u, \mu_u) \equiv\\ &\quad (position = back \supset (0, 0.25) \vee (1, 0.75) \vee (2, 0.75) \vee (3, 0.25)) \vee\\ &\quad (position = middle \supset (3, 0.25) \vee (4, 0.75) \vee (5, 0.75) \vee (6, 0.25)) \vee\\ &\quad (position = front \supset (6, 0.25) \vee (7, 0.75) \vee (8, 0.75) \vee (9, 0.5))\end{aligned}$$

while the distances can take the values

$$\begin{aligned}&\mathfrak{F}(distance, u, \mu_u) \equiv\\ &\quad (distance = very\ close \supset (0, 0.75) \vee (1, 0.25)) \vee\\ &\quad (distance = close \supset (0, 0.25) \vee (1, 0.75) \vee (2, 0.75) \vee (3, 0.25)) \vee\\ &\quad (distance = medium \supset (3, 0.25) \vee (4, 0.75) \vee (5, 0.75) \vee (6, 0.25)) \vee\\ &\quad (distance = far \supset (6, 0.25) \vee (7, 0.75) \vee (8, 0.75) \vee (9, 0.25)) \vee\\ &\quad (distance = very\ far \supset (8, 0.25) \vee (9, 0.75))\end{aligned}$$

Now that we have linguistic terms for position and distance, we want to evaluate the robot's position and its distance to the ball which is located at position 9. Suppose the robot's position in situation $S_0$ is characterized by the linguistic term *back*, i.e. $\mathcal{D}_{S_0} = \{pos(S_0) = cog'(back), ballPos(S_0) = 9\}$. Now, assume a functional fluent *dist* which returns the distance between the robot and the ball. It thus holds in $S_0$ that $\mathcal{D} \models \text{is}(dist(S_0), far)$. Now the robot travels to a position in the middle of its world. It does it by performing the action $go(4)$, meaning that the robot travels four positions to the right. Its position is updated according to the successor state axiom: $pos(do(a,s)) = y \equiv a = go(d) \wedge y = pos(s) + d \vee a \neq go(d) \wedge y = pos(s)$. This means that

$$\mathcal{D} \models \text{is}(pos(do(go(4), S_0), middle) \wedge \text{is}(dist(do(go(4), S_0), medium).$$

It holds because $pos(do(go(4), S_0)) = 5$ and $\mathfrak{F}(middle, 4, 0.75)$ has a non-zero membership value. The second part of the proposition holds because the robot is located in a middle position of the world. According to $cog'(c)$, the quantitative value for $pos(do(go(4), S_0) = middle$ is $cog'(middle) = 4$. Now the quantitative distance from 4 to 9 equals 5 units or *medium* distance, as is given by $\mathfrak{F}(medium, 5, 0.75)$. Suppose now, that the robot's control program contains the action *go*(*far*) mentioning the qualitative term *far*. At which position will the robot end up in situation $s = do(go(far), S_0)$? The successor state axiom for the position fluent does not handle qualitative categories by now. It is, though, possible to treat all different qualitative categories in the successor state axiom. For simplicity, we assume, that we apply the function $cog'(c)$ to qualitative terms which yields always a quantitative representative. This way, we do not need to distinguish between qualitative and quantitative values in the successor state axioms. Assume further that the initial position is given by the qualitative term *back*, i.e. $pos(S_0) = cog'(back)$. Then, $\mathcal{D} \models \text{is}(pos(do(go(cog'(far)), S_0)), front)$ if and only if $\mathcal{D}_{S_0} \models \text{is}(\mathcal{R}[pos(do(go(cog'(far)), S_0))], front)$ which is regressed to $pos(S_0) = cog'(back) \wedge \text{is}(y = cog'(back) + cog'(far), front) \equiv \text{is}(y = 1 + 7, front) \equiv y = 8 \wedge \mathfrak{F}(front, 8, 0.75) \wedge 0.75 > 0$. Thus, it can be inferred that the robot will end up at position *front*. For the regression operator $\mathcal{R}$ which replaces a fluent in situation $do(a,s)$ with the right hand side of the successor state axiom we refer to [19].

## 5 Conclusion

In this paper we formalized a novel semantics for qualitative fluents in the situation calculus. Each quantitative fluent value is associated with a membership value, describing to which degree the respective fluent value belongs to the qualitative category. Therefore, for each qualitative category, one has to specify pairs $(u, \mu_u)$ defining to which degree $\mu_u$ the fluent value $u$ belongs to a particular category. Further, one needs to be able to query, if a fluent value belongs to a qualitative category. Therefore, we introduced the predicate *is* which is true iff a fluent $\mathfrak{f}$ belongs to the category $c$. Finally, we introduced an assignment operator which allows to assign a qualitative category to a fluent. As is done in fuzzy control theory, the semantics of this operation is *defuzzification*, i.e., we assign a single quantitative value as a representative for the whole qualitative category. With this, we have now a defined semantics for qualitative values in the situation calculus. Although it was not in the primary focus of this paper, we can now easily interpret fuzzy rules of the kind "if $A$ is $R_1$ and $\cdots$ and $A$ is $R_n$ then $Y{:=}B$". While this work lays the theoretical foundation, in our future work we will develop robot controllers making use of these qualitative fuzzy predicates. As another future step, we are going to formalize other fuzzy inference rules following Mamdani, Surgeno, or the gradual rules method (see e.g. [5]).

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